CADLM

Explore new industrial horizons

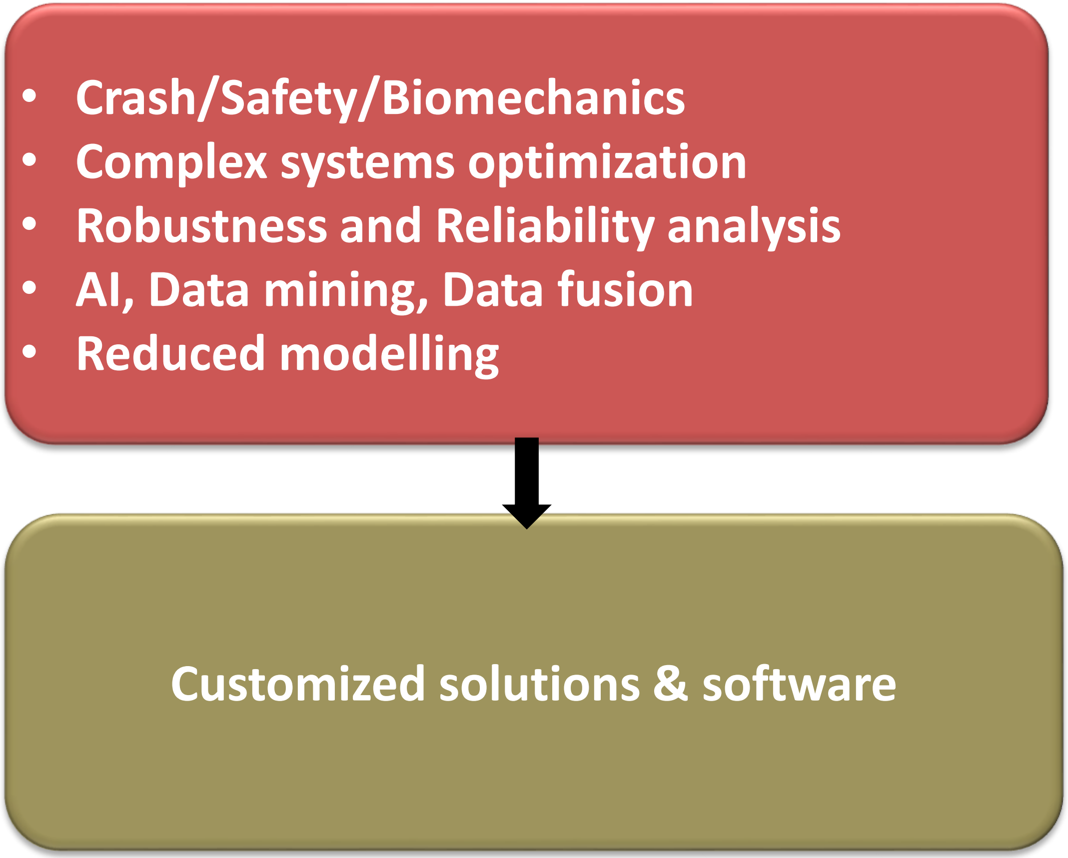
Optimal Decision Support System for Engineering and Expertise

La réduction de modèles : une solution efficace pour la modélisation paramétrique, optimisation et analyse de la robustesse

Model reduction : An efficient solution for parametric analysis, optimization, robustness and reliability analysis

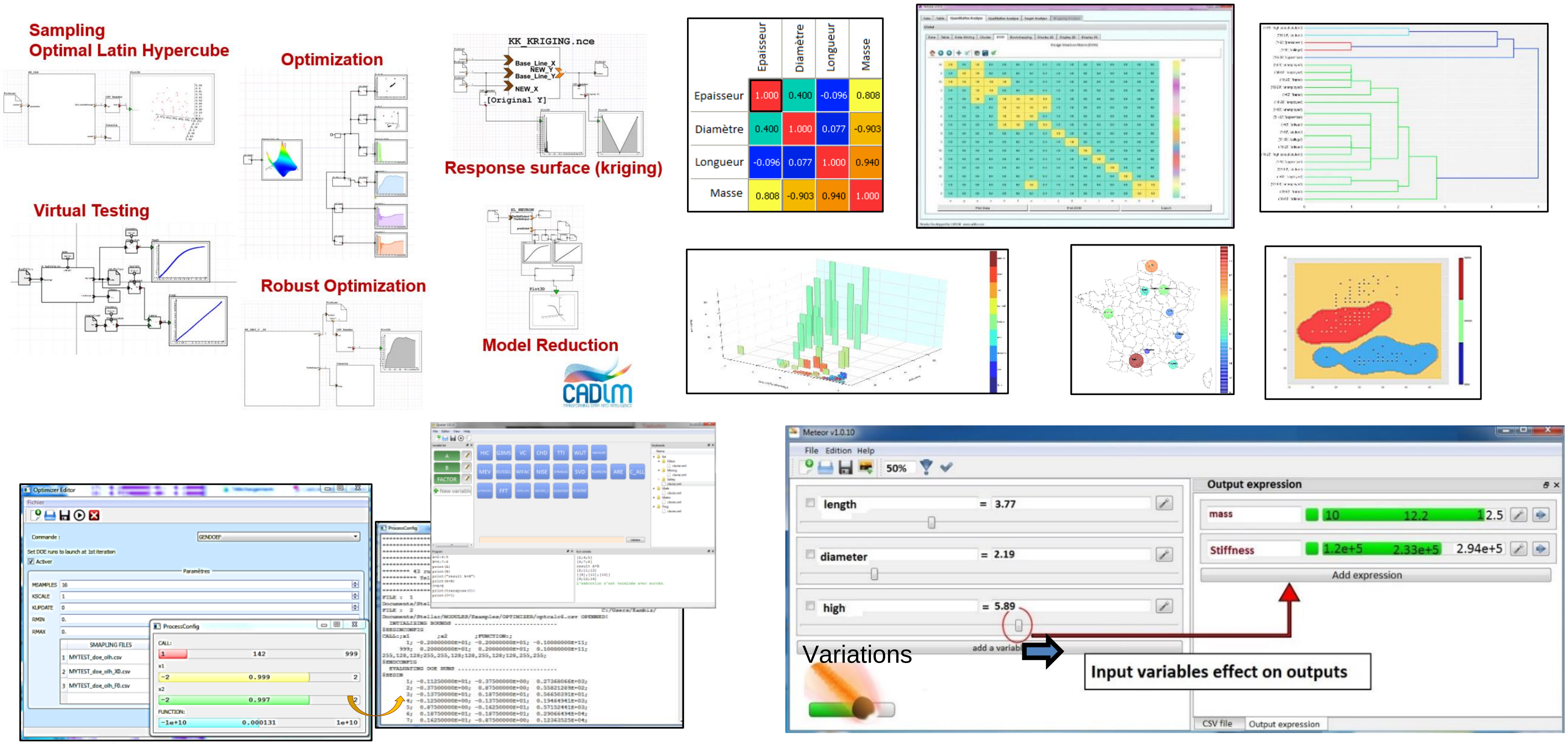
Kambiz Kayvantash : kambiz.kayvantash@cadlm.com

CADLM: Transforming data into intelligence



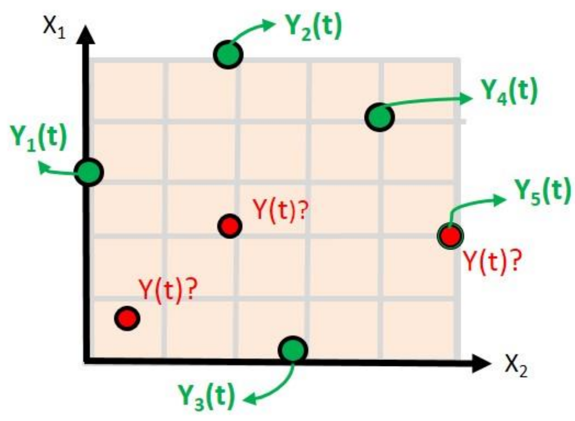
Quasar — Calculator for data mining and performance indicators
Nova — System modelling & optimization
Nebular — Industrial & business analitic
Meteor — Visual modeling interpolation
Stellar — The process chain builder
Pulsar — Simple file or data base connection

ODYSSEE



	Epaisseur	Diamètre	Longueur	Masse
Epaisseur	1.000	0.400	-0.096	0.808
Diamètre	0.400	1.000	0.077	-0.903
Longueur	-0.096	0.077	1.000	0.940
Masse	0.808	-0.903	0.940	1.000

ODYSSEE



Y(t) inclus dans la base de travail , Y(t) à prédire.

Lunar
Data fusion & model reduction

Optimisation rapide des systèmes

physiques et numériques

grâce à la prédiction instantanée

(application embarquée)

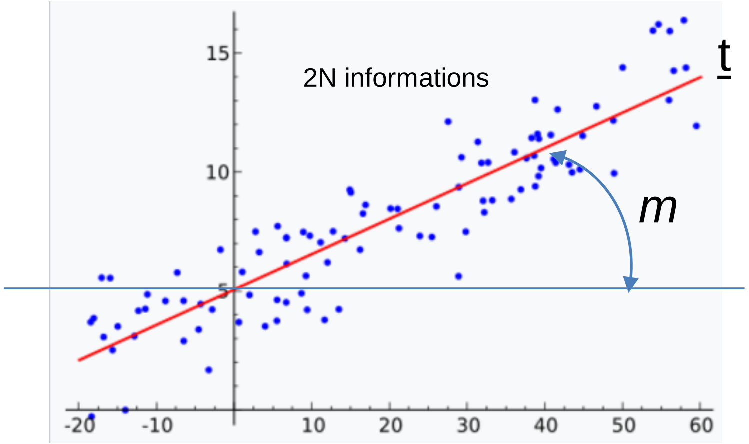
Predicting impact of new parameter sets without FE computing or additional experiments.

Model Reduction
(ROM=Reduced Order Modelling)

- Introduction on Modal Decomposition
- PGD (Proper Generalized Decomposition)
- Hyper-Reduction
- **POD (Proper Orthogonal Decomposition)**
- Reduced basis methods, Polynomial Chaos (Lagrange, Hermit, Taylor)
- **CVT (Centroidal Voronoi tessellations)**
- **Applications Crash/Safety**

Example - Equation of straight line

Slope, intercept : $y = m\,x + b.$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ → 2 or 5



Point, intercept : $y - y_1 = m\,(x - x_1).$ → 3

General form $a\,x + b\,y + c = 0.$ → 3

Parametric form

$$x = x_1 + t$$
$$y = y_1 + m\,t$$

Each value of t gives a different point on the line.

→ 3

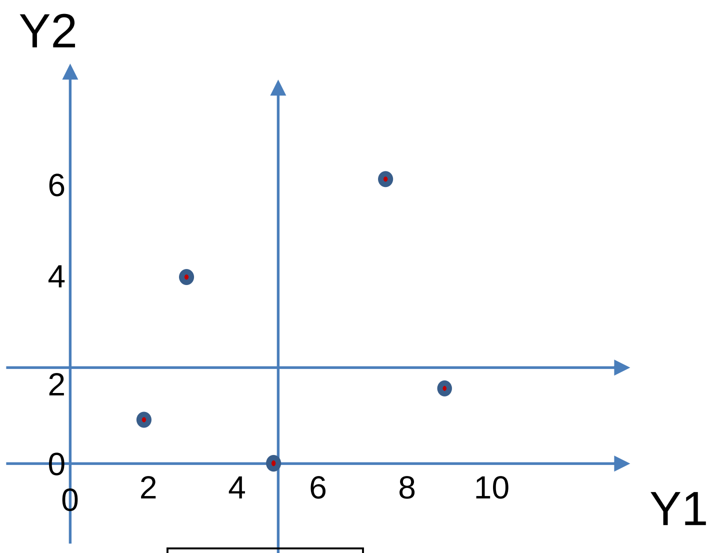
(eigenvalue -> eigenvector)

Example - Equation of straight line

Parametric form	$x = x_1 + t$ $y = y_1 + m\,t$	Each value of t gives a different point on the line.

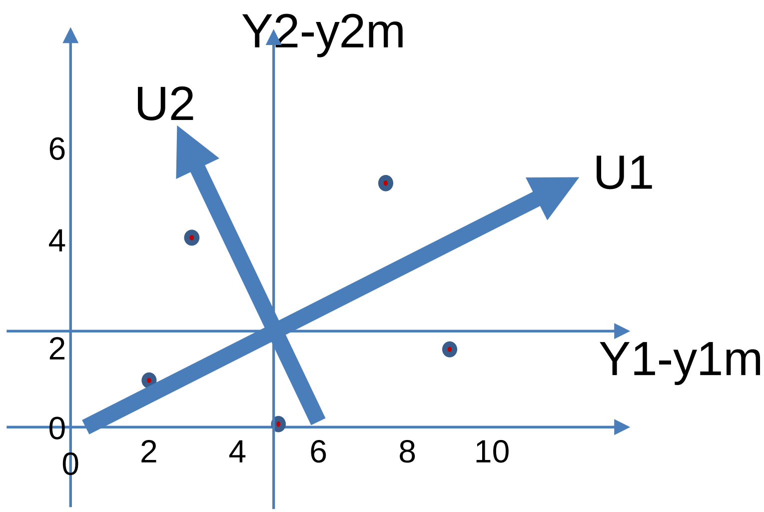
- Information is reduced/ regressed / fusioned
- Instead of 2N data we need only need N+4 ($\underline{t}_N$, m, x_1, y_1)
- $\underline{\boldsymbol{t}}$ is an **eigenvector** and ***m*** is an **eigenvalue**
- We can reconstruct (approximately) **X** and **Y** knowing only $\underline{\boldsymbol{t}}$ and ***m*** !

PCA



Y=
2 ; 1
3; 4
5 ; 0
7 ; 6
9 ; 2

Strong coupling



Y=
-3.578 ; 0
-1.342 ; 2.236
-1.342 ; -2.236
3.130 ; 2.236
3.130 ; -2.236

Weak coupling

Modal Analysis

$$M\ddot{a} + C\dot{a} + Ka + f = 0$$

Strong coupling

$$a = \bar{a}e^{\alpha t} = \sum_{i=1}^{n} \bar{a}_i \, e^{\alpha_i t}$$

Transformation

$$m_i \, \ddot{y}_i + c_i \dot{y}_i + k_i y_i + f = 0$$

Weak coupling

$$m_i = \bar{a}_i^T \, M \, \bar{a}_i = 1 \; (if \; modes \; are \; normalized)$$

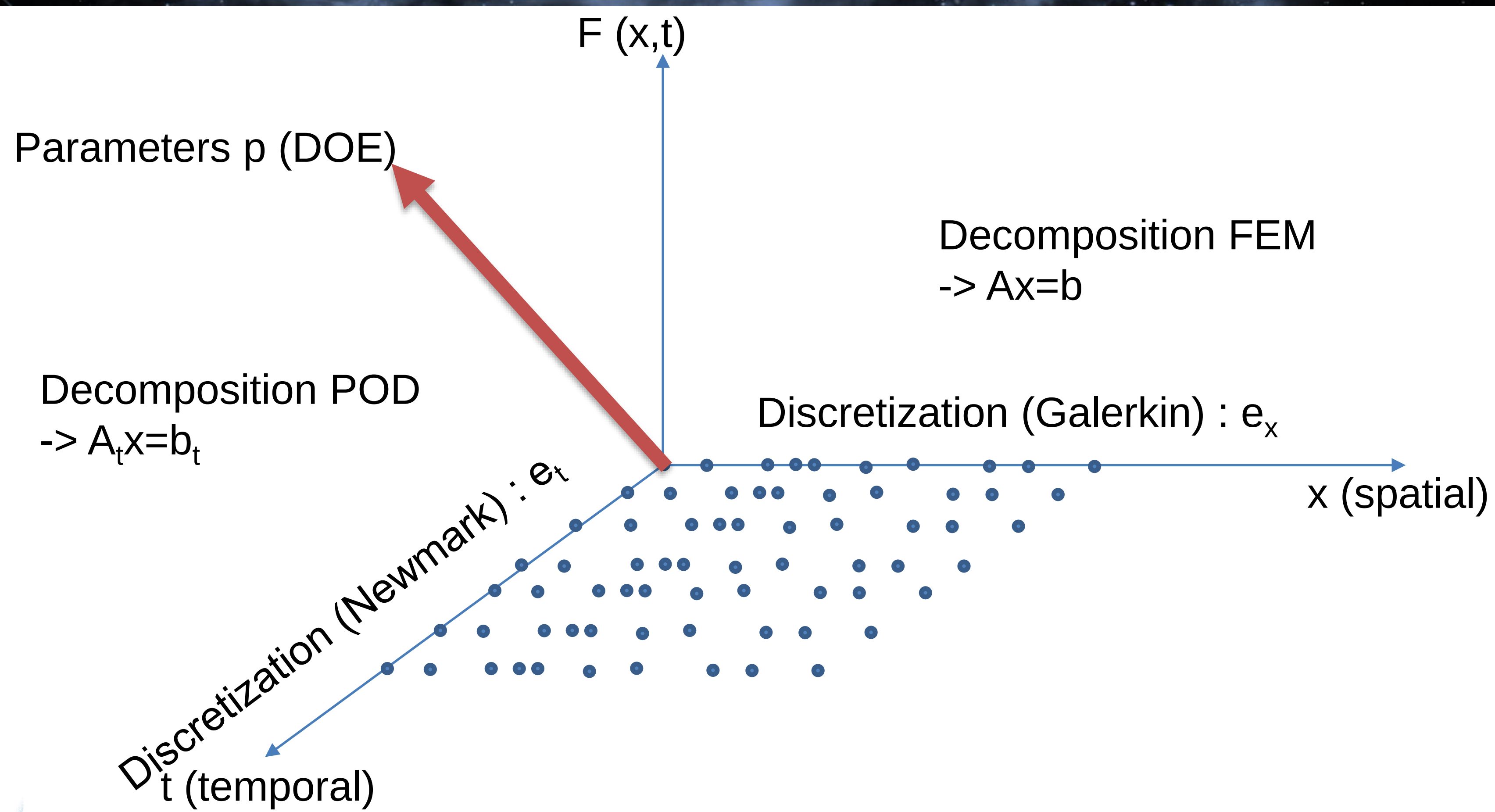
$$c_i = \bar{a}_i^T \; C \, \bar{a}_i$$

$$k_i = \bar{a}_i^T \; K \, \bar{a}_i$$

$$f_i = \bar{a}_i^T \; f$$

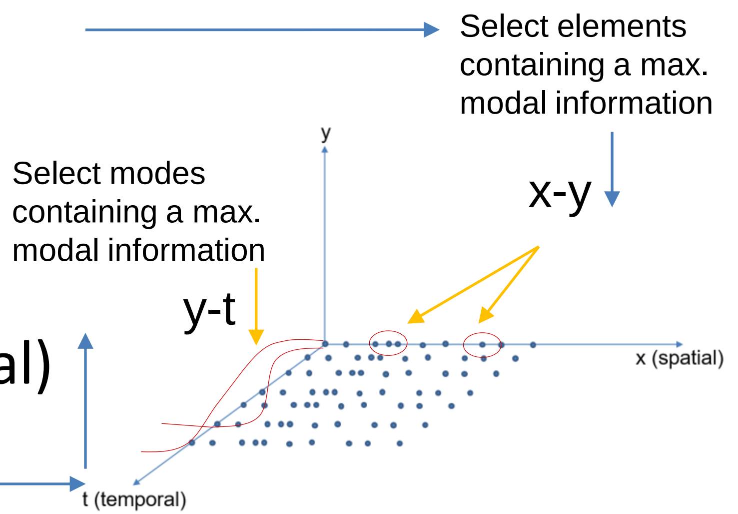
FE versus ROM

y



Model reduction techniques

- **Intrusive**
  - A priori, Solver dependent
    - Shape functions (elements, spatial)
    - Reduced stiffness matrix (elements, spatial and temporal)
- **Non-Intrusive**
  - A posteriori, post-processing



PGD (Proper Generalized Decomposition)

- **PGD (proper generalized decomposition)** is an approximation of the weak solution of parametric FE model
- Construction of tensorial formalism of the Newmark equations for whole of time domain at once in order to obtain a new system of linear equations
- Separation of variables in spatial and temporal modes
  - Y(x,t) = Σ G(x) . H(t)
- Application of time-series approach in temporal domain (iterative)

$$\Rightarrow \boldsymbol{x}(t, \boldsymbol{\Delta}) = \sum_{i=1}^{\infty} \Phi_i(\boldsymbol{\Delta}) \mathcal{X}_i(t) \tag{1}$$

where $\Phi_i(\boldsymbol{\Delta}) \in (\mathbb{R}, \mathcal{B}(\mathbb{R}))$, and $\mathcal{X}_i(t) \in \mathbb{R}^m \; \forall \, i = 1, \ldots, n$. If we truncate the sum to N terms, we have,

$$\boldsymbol{x}^N(t, \boldsymbol{\Delta}) = \sum_{i=1}^{N} \Phi_i(\boldsymbol{\Delta}) \mathcal{X}_i(t), \tag{2}$$

In PGD framework, the sum $\boldsymbol{x}^N(t, \boldsymbol{\Delta})$ is constructed incrementally by solving for the functions $\Phi_i(\boldsymbol{\Delta})$ and $X_i(t)$. We assume that the sum given in (2) is known upto $n < N$ terms. Next we introduce a candidate $n+1^{\text{th}}$ term such that

$$\boldsymbol{x}^{n+1}(t, \boldsymbol{\Delta}) = \boldsymbol{x}^n(t, \boldsymbol{\Delta}) + P(\boldsymbol{\Delta}) \boldsymbol{Y}(t). \tag{3}$$

where $P(\boldsymbol{\Delta}) \in (\mathbb{R}, \mathcal{B}(\mathbb{R}))$ and $\boldsymbol{Y}(t) \in \mathbb{R}^m$ are unknown functions. The couple $P(\boldsymbol{\Delta})$ and $\boldsymbol{Y}(t)$ is known as the *enrichment couple*. We then solve for $P(\boldsymbol{\Delta})$ and $\boldsymbol{Y}(t)$. This *enrichment procedure* continues until,

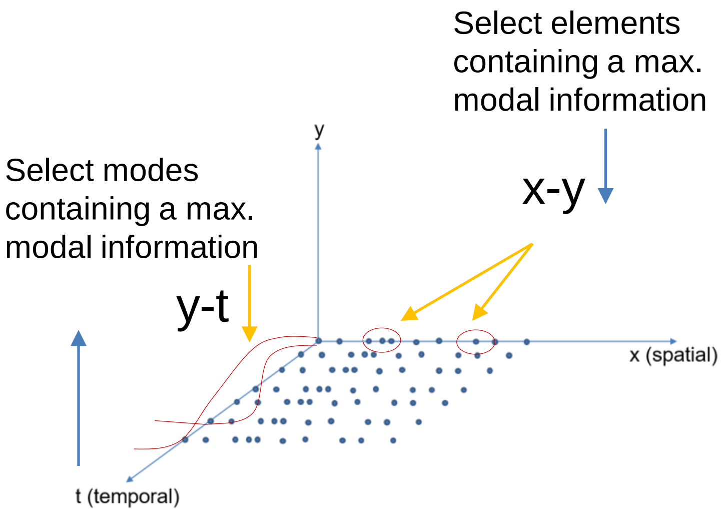
$$\|\boldsymbol{x}^{n+1}(t, \boldsymbol{\Delta}) - \boldsymbol{x}^n(t, \boldsymbol{\Delta})\| < \epsilon$$

Reduced Order Modeling for Systems with Parametric Uncertainty Using Proper Generalized Decomposition
Parikshit Dutta

See also Chinesta et al.

Intermediate methods

- **Hyper-reduction**
  - An adaptive modification of the base as we go on (in time) which necessitate a new basis reconstruction
  - Adaptive meshing (use POD between two re-meshing FE steps) an initial DOE anyway (Full FE) -> POD ? (needs initial basis)



See also David Ryckelynck

POD
for on-board and real-time computing

- **POD (Proper orthogonal decomposition)** are algebraic approximation solutions allowing for fast (real-time) interpolations
- **Reconstructions or extrapolations** based on previously existing DOE-type results (snapshots) obtained either from FE computations or directly from experimental results
- **Unlike response surface methods** where smoothed solutions on certain criteria are obtained, **POD provides complete solutions** (reconstructions) of the space-time response of the original differential equation.

$$\mathbf{F(x,t) = G(x) \,.\, H(t)}$$

POD

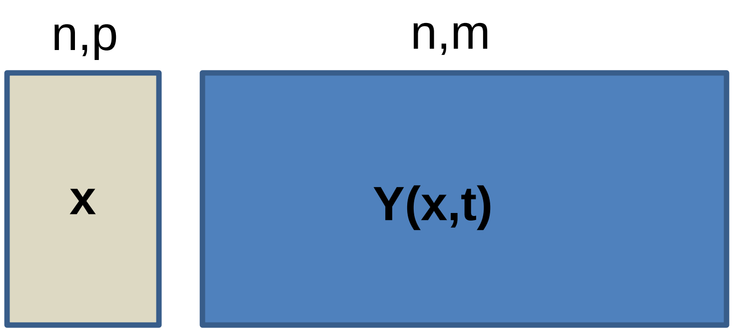
- Let X $\{x_1,\ldots..,x_p\}$ and Y $\{y_1,\ldots..,y_m\}$
- Assume $y_k = \hat{A}\, x_p$ -> find $\hat{A}$; *(p =< m)*
- Decompose $Y_{n,m}$
  - **If m>>n -> Direct method (SVD)**
  - If m<<n -> Snapshots method (Correlation Matrix)

$$M\ddot{a} + C\dot{a} + Ka + f = 0$$

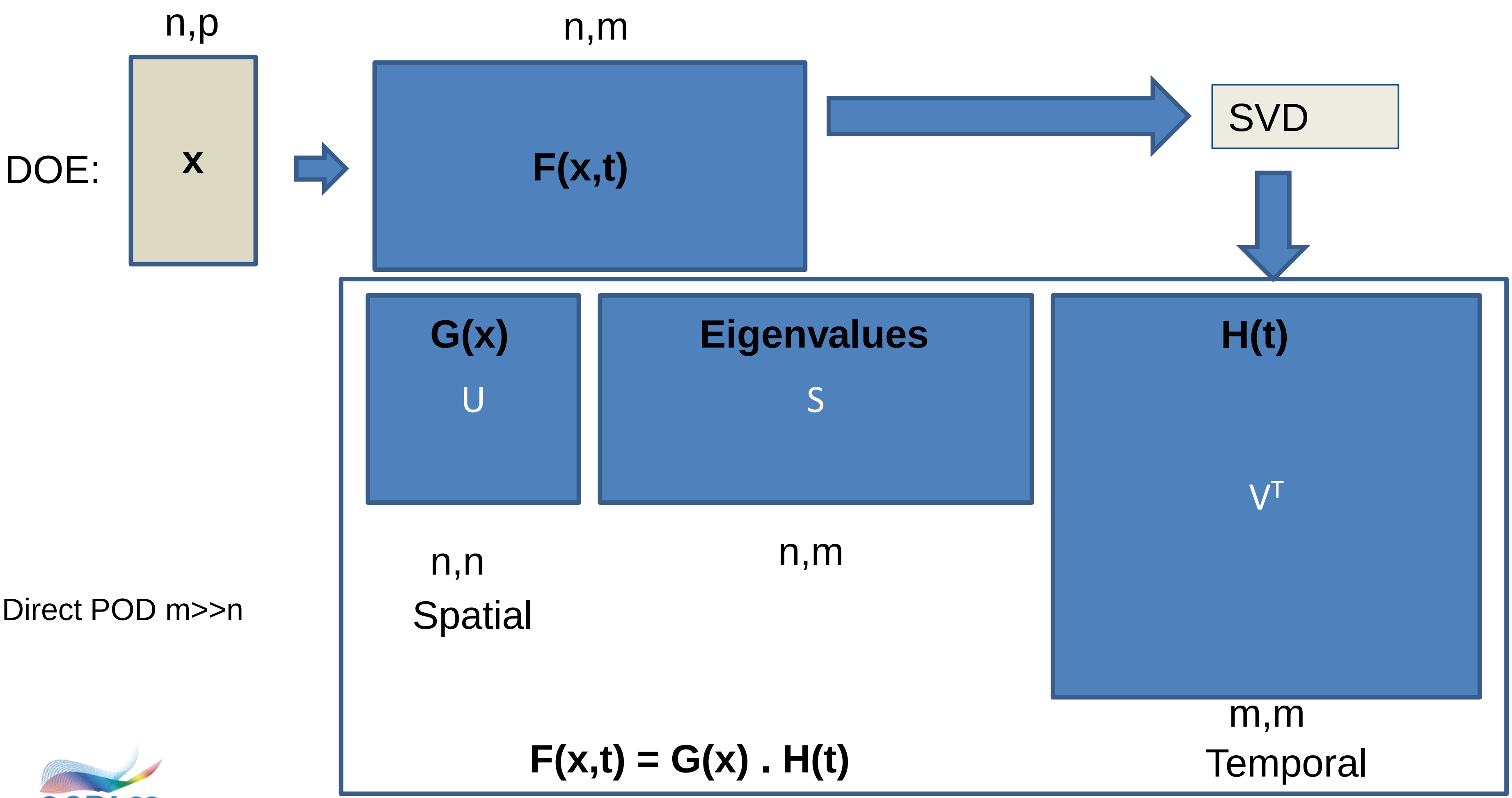
$$\Downarrow$$

$$a = \bar{a}e^{\alpha t} = \sum_{i=1}^{n} \bar{a}_i\, e^{\alpha_i t}$$

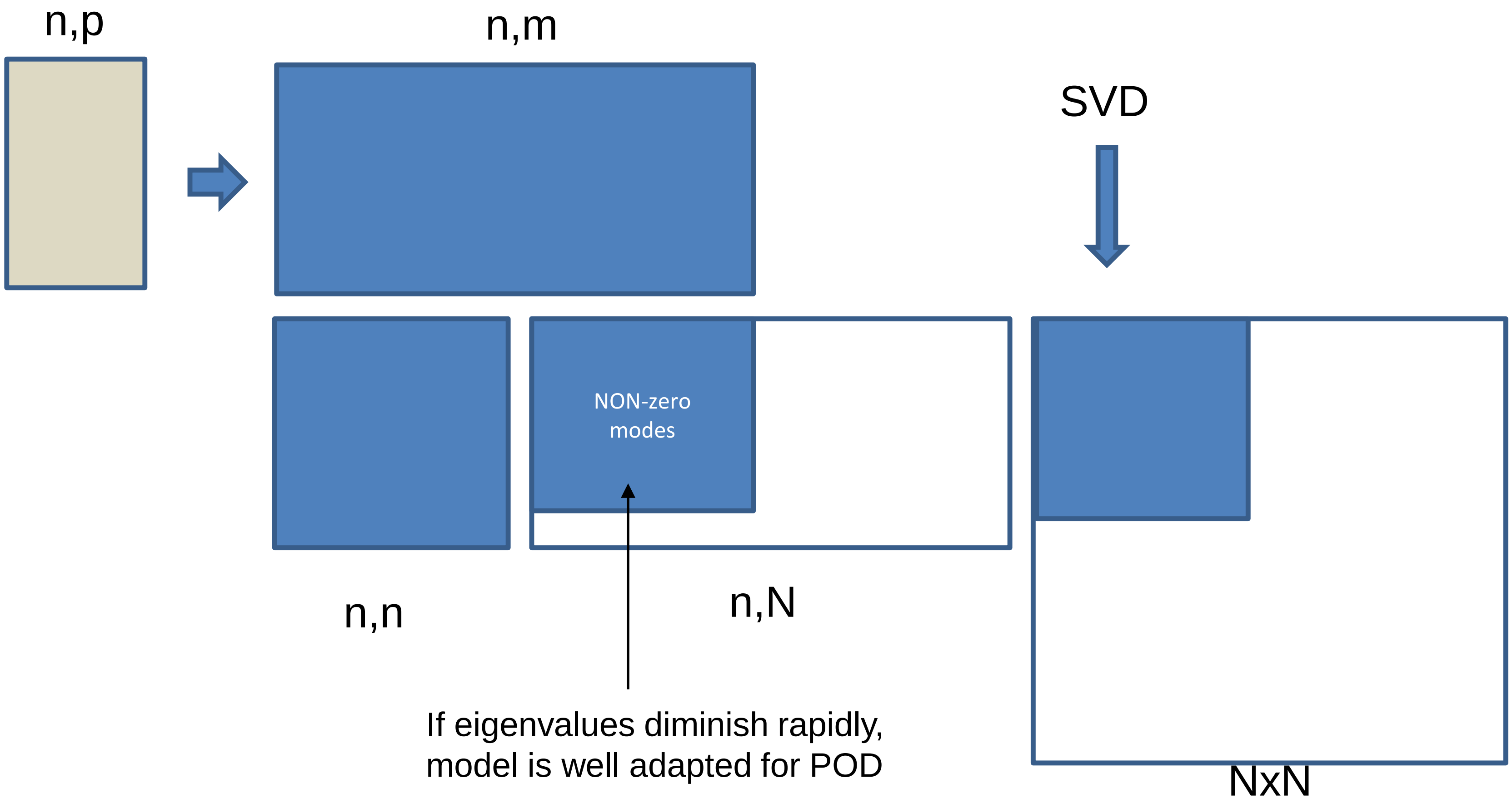
Modal basis
(c.f. PCA)



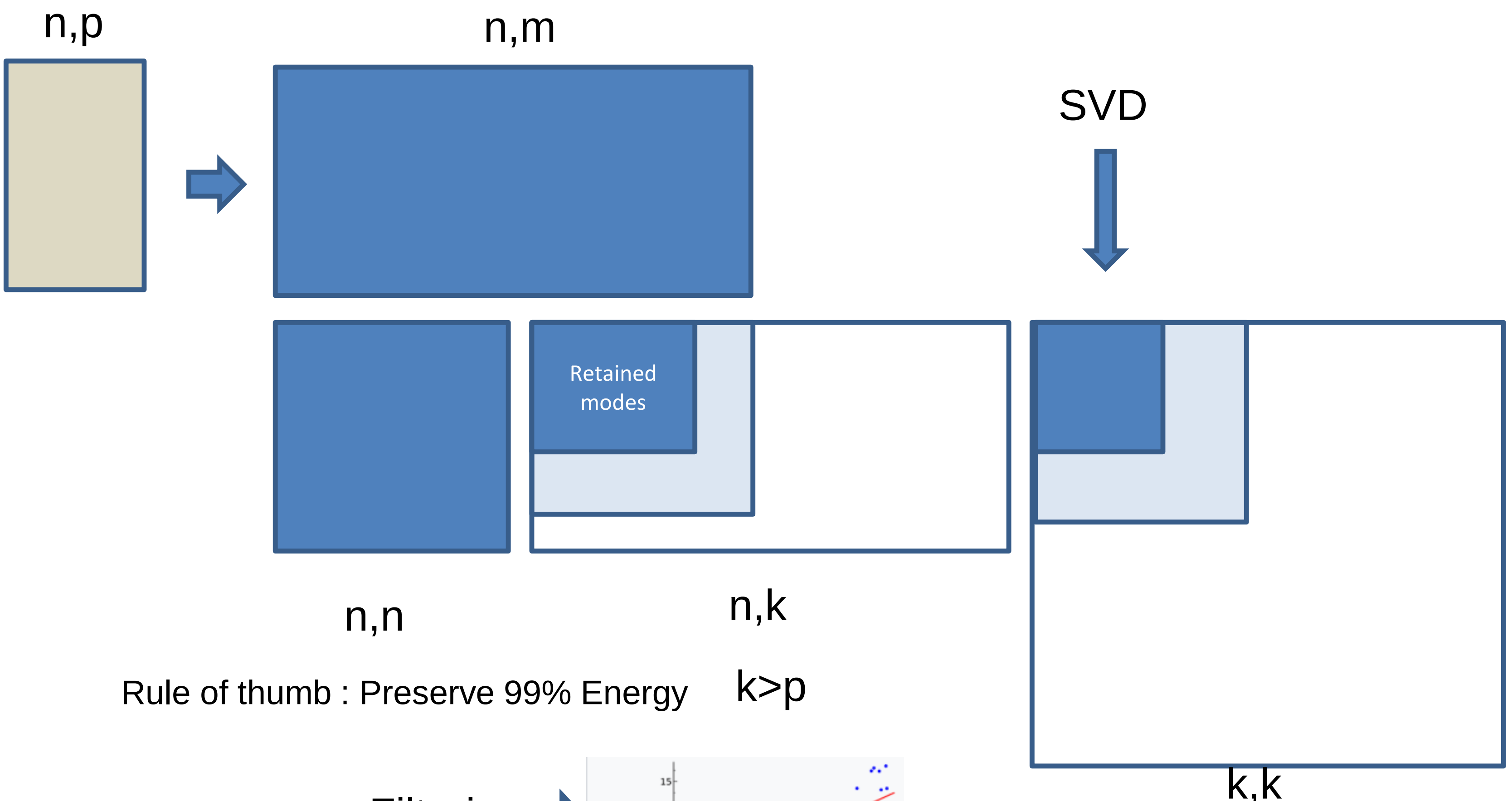
Singular Value Decomposition



Singular Value Decomposition

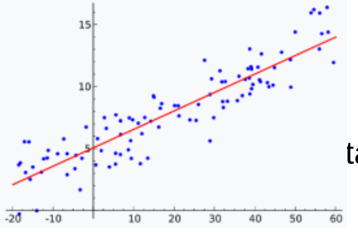


Singular Value Decomposition



Rule of thumb : Preserve 99% Energy

Filtering



Summary of a POD algorithm

1) EF → $\mathbf{Q}(\mathbf{x}\ ,\ t)\quad (t_0<t<t_N)$

2) DOE → $\mathbf{x}_{(n,\ p)} : \mathbf{Q}_{(n,\ m)}$

3) SVD → $\mathbf{Q} = \mathbf{U}_{(n,\ n)}\ \mathbf{S}_{(n,\ m)}\ \mathbf{V}^{\mathbf{T}}_{(m,\ m)}$

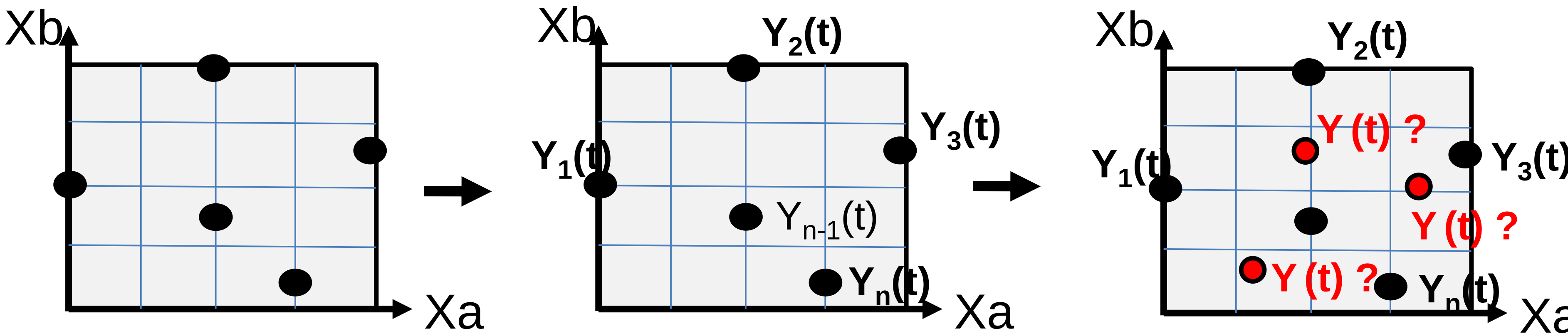
4) RDA → $\mathbf{x}\ |\ \mathbf{U}\ \mathbf{S}\quad (O(\mathbf{x})=\mathbf{q}(\mathbf{x}))$

5) $O(\mathbf{x}_{new})$ → $\mathbf{q}_{new}$

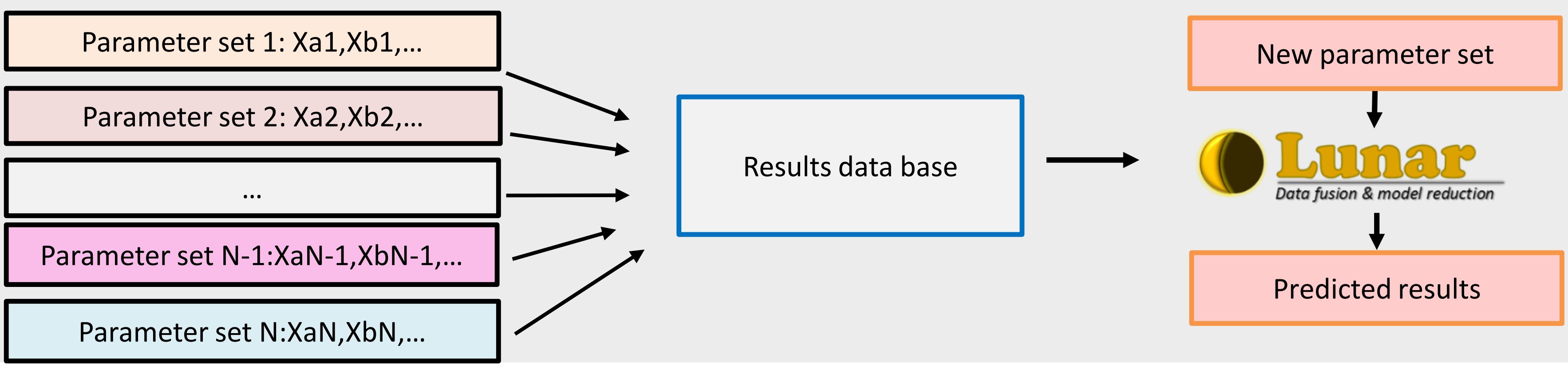
6) $\mathbf{q}_{new} \cdot \mathbf{V}_T = \mathbf{Q}(\mathbf{x}_{new}, t_N)$

Know-how

How it works

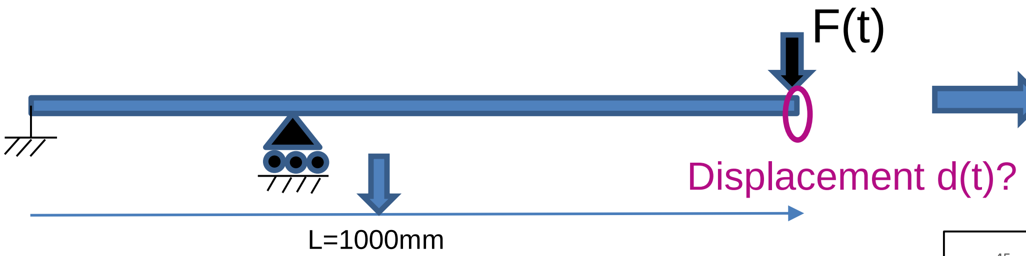


Xb
Xa
Xb
Xa
$Y_2(t)$
$Y_3(t)$
$Y_1(t)$
$Y_{n-1}(t)$
$Y_n(t)$
Xb
Xa
$Y_2(t)$
$Y_3(t)$
$Y_1(t)$
$Y_n(t)$
Y (t) ?
Y (t) ?
Y (t) ?

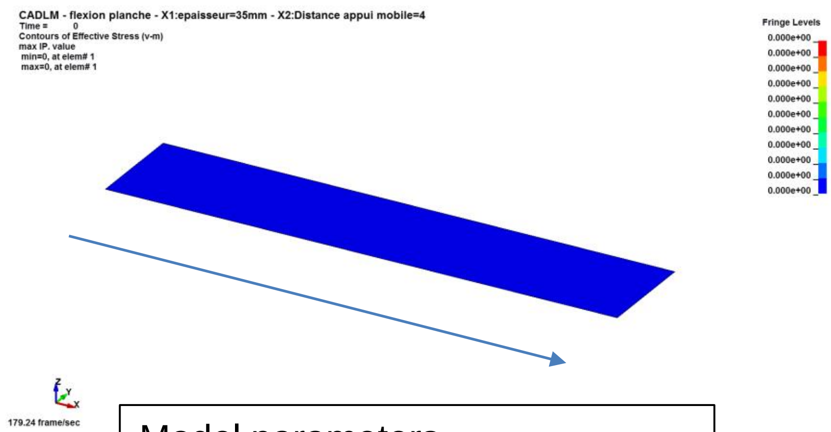


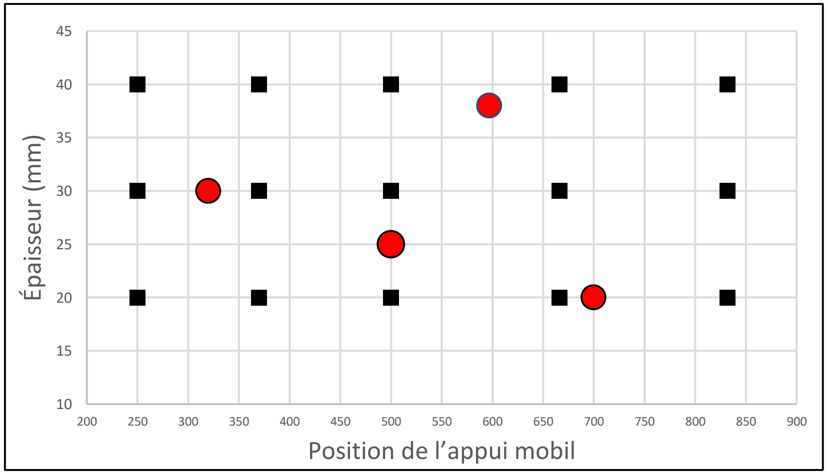
Parameter set 1: Xa1,Xb1,...
Parameter set 2: Xa2,Xb2,...
...
Parameter set N-1:XaN-1,XbN-1,...
Parameter set N:XaN,XbN,...
Results data base
New parameter set
Lunar
Data fusion & model reduction
Predicted results

Example – plate flexion



Influence lines

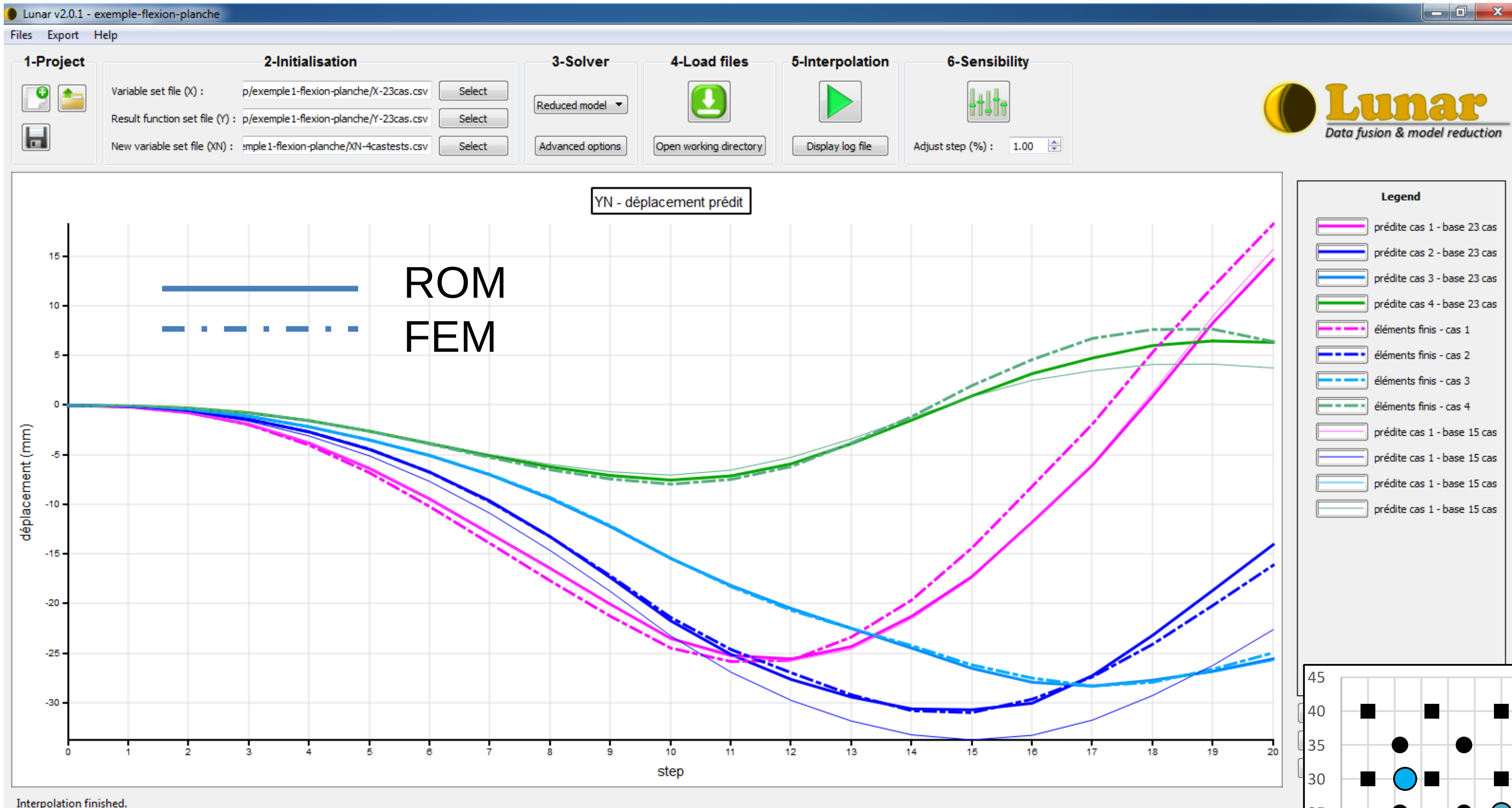


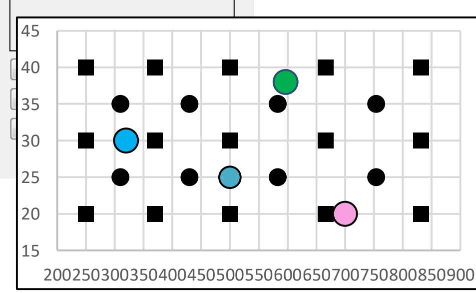


Model parameters
X1: plate thickness
X2: mobile support distance

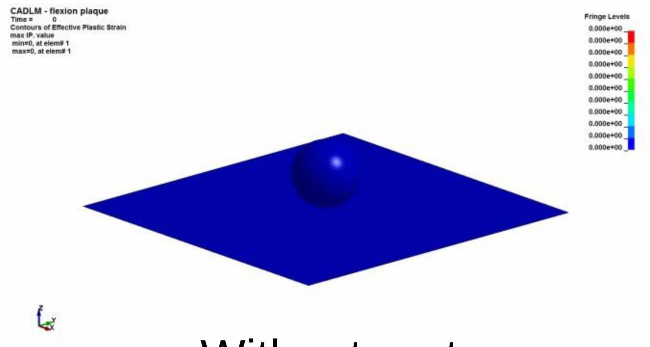
Predict: Y: displacement-time over the complete spatial domain

Example – plate flexion

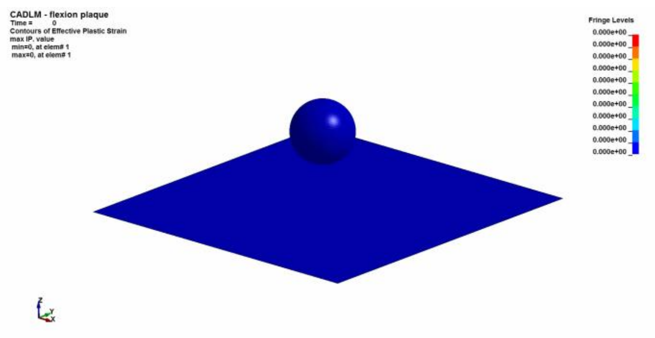




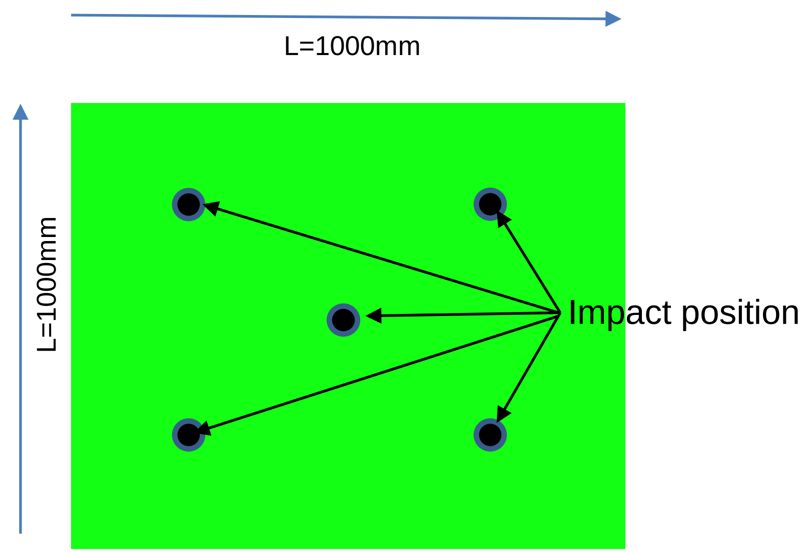
Example – plate impact



Without rupture



With rupture

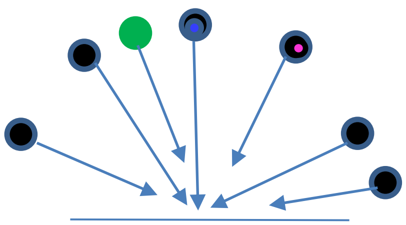


Model parameters:
X1: impactor velocity
X2: impact position
X3: impact angle

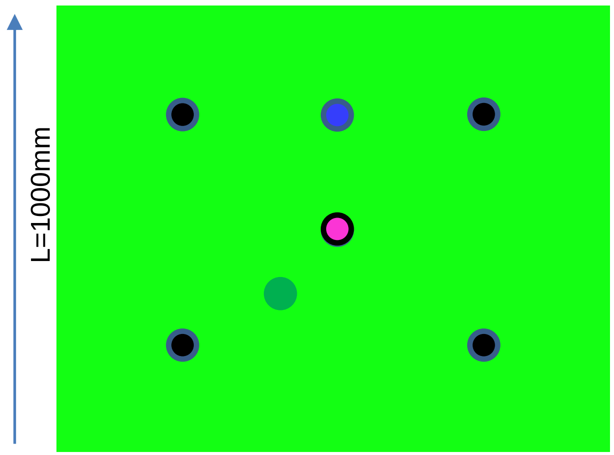
Predict:
Y: internal energy
Y: impact force

Example – plate impact

Model parameters:
X1: impactor velocity
X2: impact position
X3: impact angle



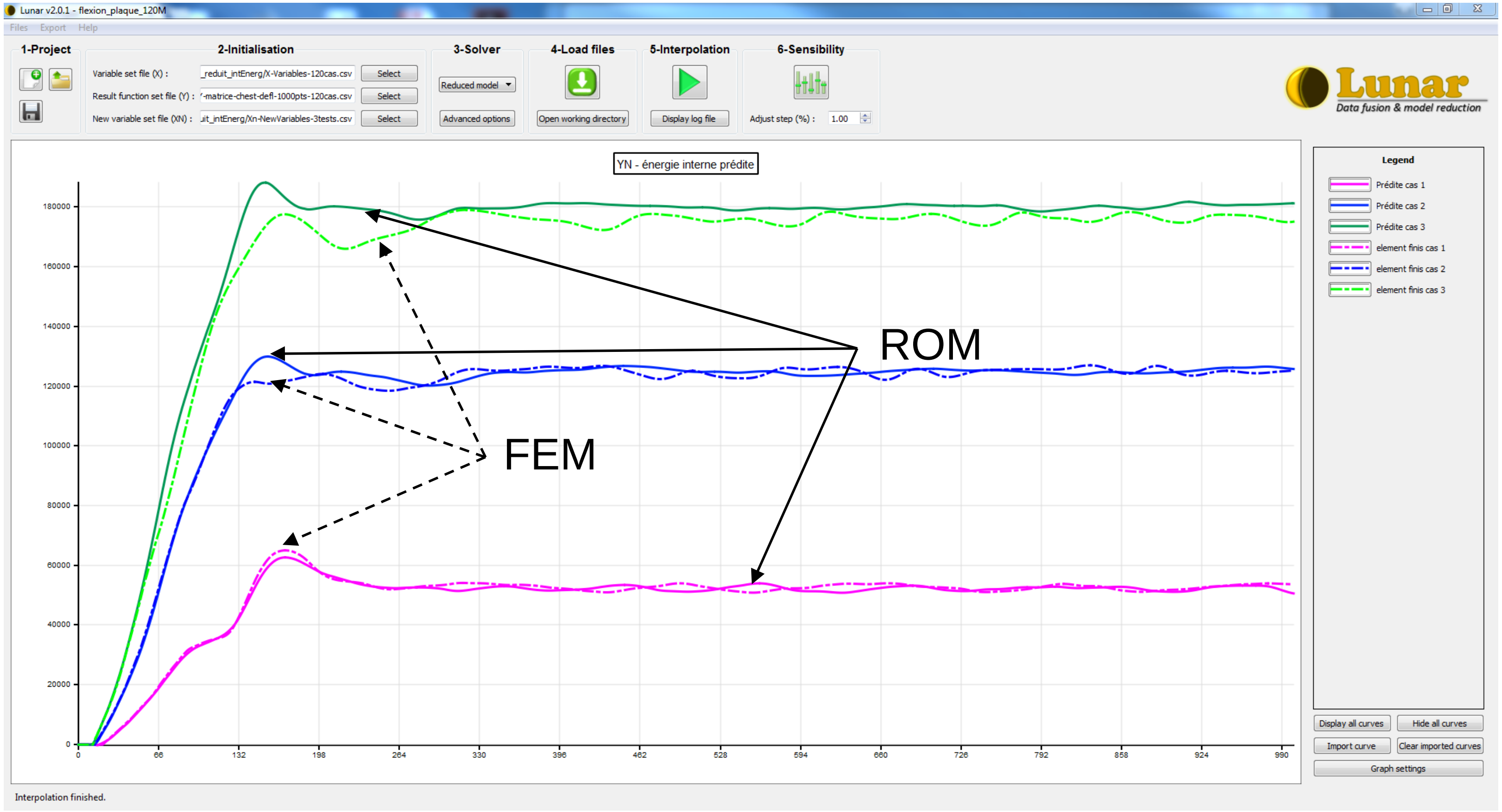
(mm/s)	V1	V2	V3	V4
DOE	5000	7500	8750	10000
Test	6000	7500	9000	



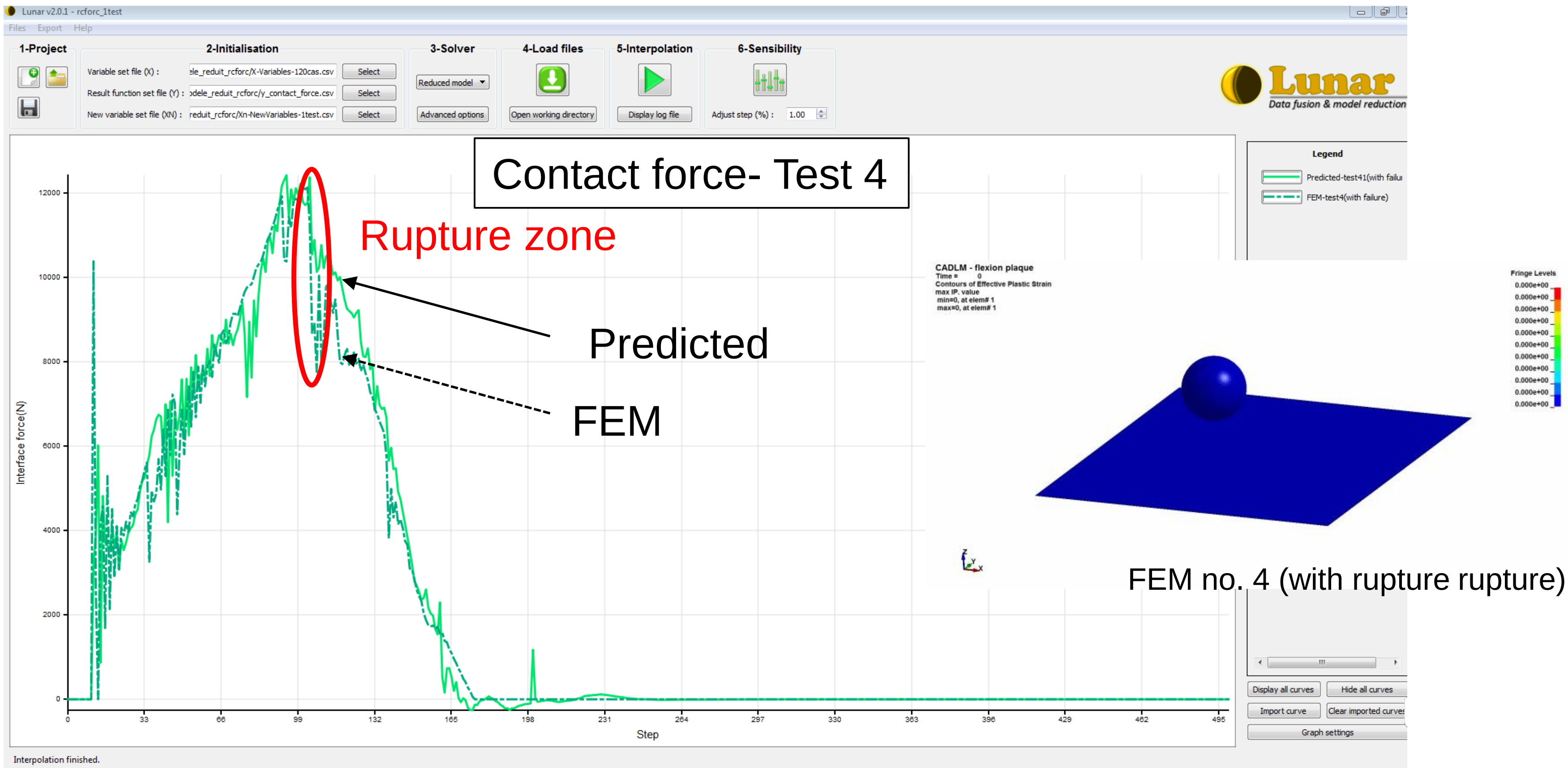
Test 1 : New velocity
Test 2 : New position
Test 3 : New velocity; New velocity; New angle

Mesure à prédire:
Y: internal energy versus time

Example – plate impact



Example – plate impact

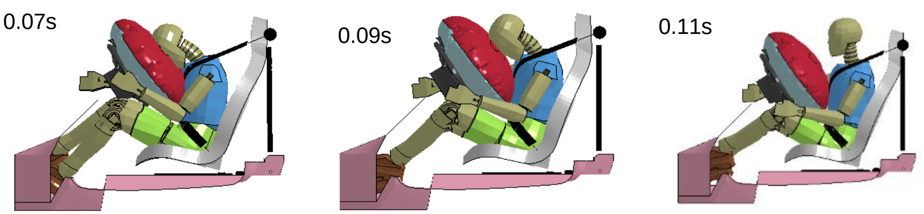


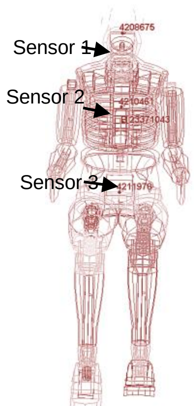
SIMBIO-M--CADLM-REDUCED MODELLING, Kambiz Kayvantash–
18/19.06.2018

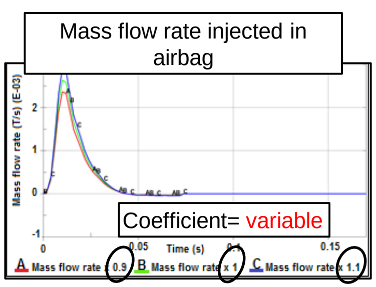
Example – Sled test

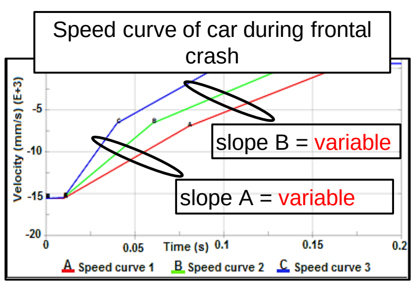
Predict head, thorax and pelvis acceleration : a(t)

Predict thorax compression: d(t)









X1: impact velocity : Slope A

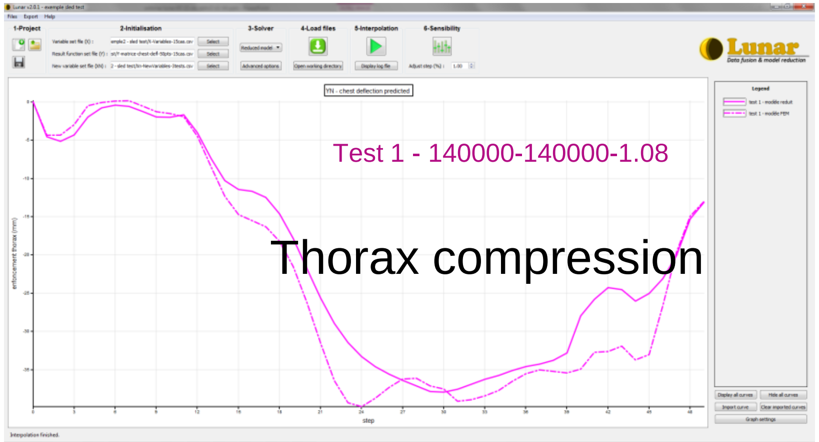
X2: impact velocity : Slope B

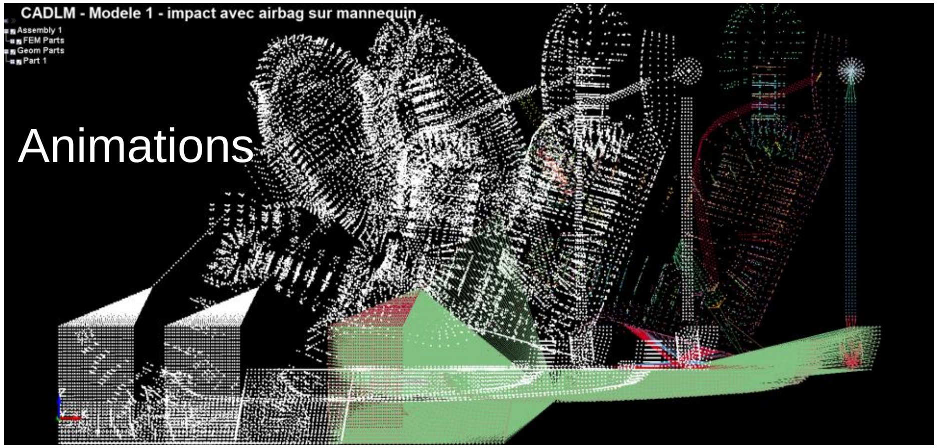
X3: injected airbag mass debit (enthalpy)

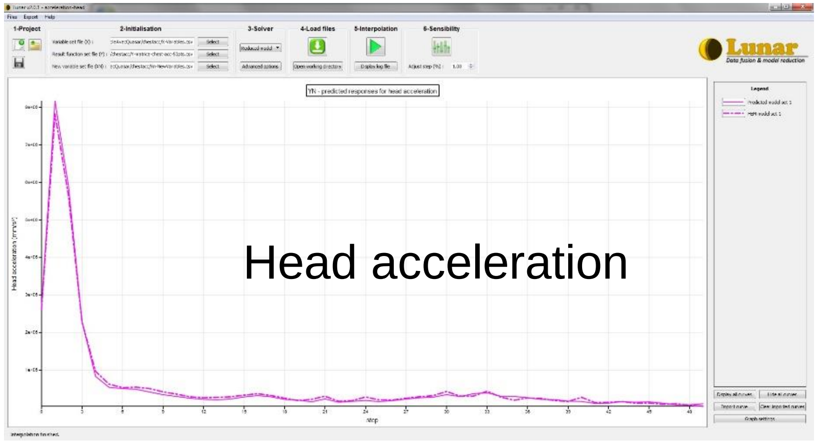
X1	X2	X3	Y1	Y2
Velocity Slope A	Velocity Slope B	Injected gas Coeff.	Chest deflection	Head Injury criteria
122285	83559	0.9	36	358
122285	83559	1	37	371
122285	83559	1.1	38	391
179679	88704	0.9	42	667
179679	88704	1	43	596
179679	88704	1.1	42	714
296105	118272	0.9	43	955
296105	118272	1	57	1076
296105	118272	1.1	58	1061
98048	189184	1.04	34.5	212
150000	310000	0.93	44.5	978
160800	62000	0.85	39	359
230000	230000	0.8	43	1413
116170	114475	0.902	34	279
189541	69353	0.84	37	420

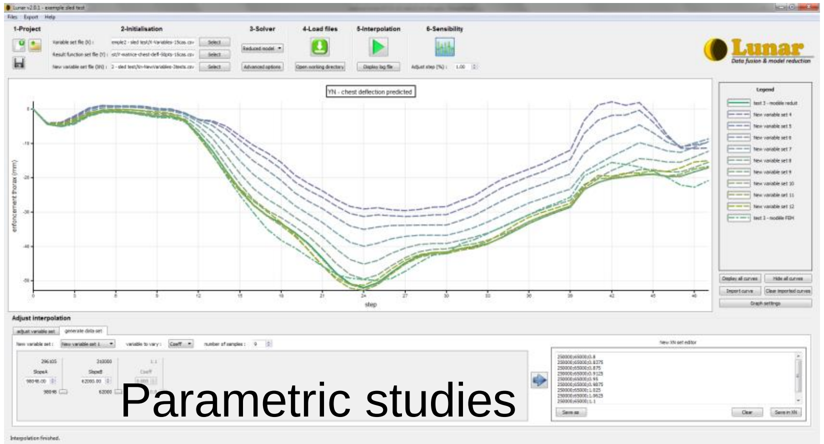
Kambiz Kayvantash, SIMBIO-M , 2015

Example – Sled test Predictions





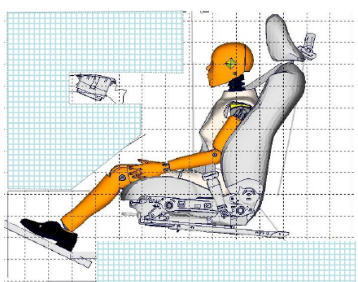




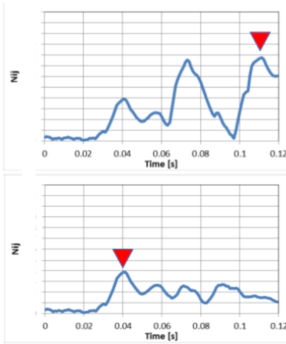
Example – Validation of Nij with HIII F50

Background

•Nij of HYBRIDⅢ AF05%ile dummy at passenger seat was studied by sled FE model. Timing of the maximum value of Nij depended on parameter of seat belt and airbag. Response surface method may not be applicable to optimize Nij in this case. ODYSSEE, developed by CADLM company and presented at 10th European LS-DYNA Conference, was applied to estimate time history of Nij.



Sled FE model for NCAP



Case A:Small vent hole with weak force limiter

Case B:Medium vent hole with weak force limiter

Time history of Nij

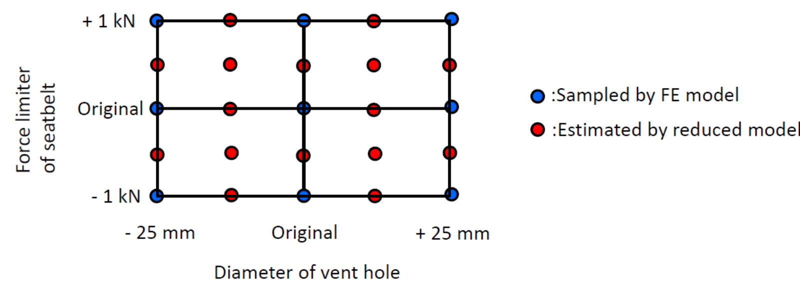
Integrated Safety TOYOTA

Case study

•Force limiter of seatbelt and diameter of vent hole were parameters.

•Nine cases were sampled and simulated by FE model for generating reduced model.

•Sixteen cases were reconstructed by the reduce model and validated with results of FE model.

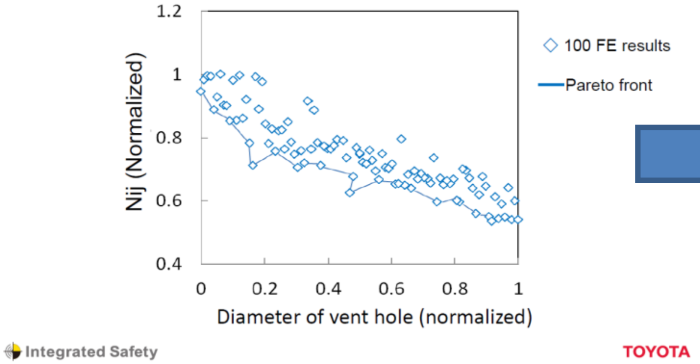


Integrated Safety TOYOTA

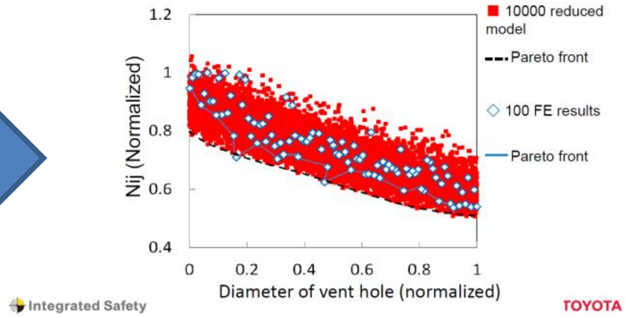
Courtesy of Toyota Motor Company – Tsuyoshi Yasuki, LS-DYNA conference 2016

Population Studies

•100 FE results seemed be not enough to identify exact Pareto front curve.

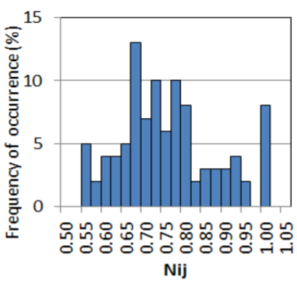


•10000 reduced model results showed smoother Pareto front curve than 100 FE results with CPU time of 10 seconds on PC.

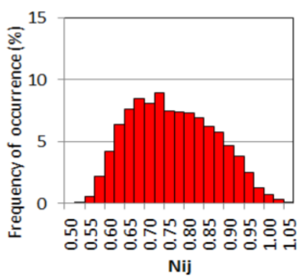


Frequency distribution of dummy injury scale

•10000 reduced model results showed more realistic frequency distribution than 100 FE results.



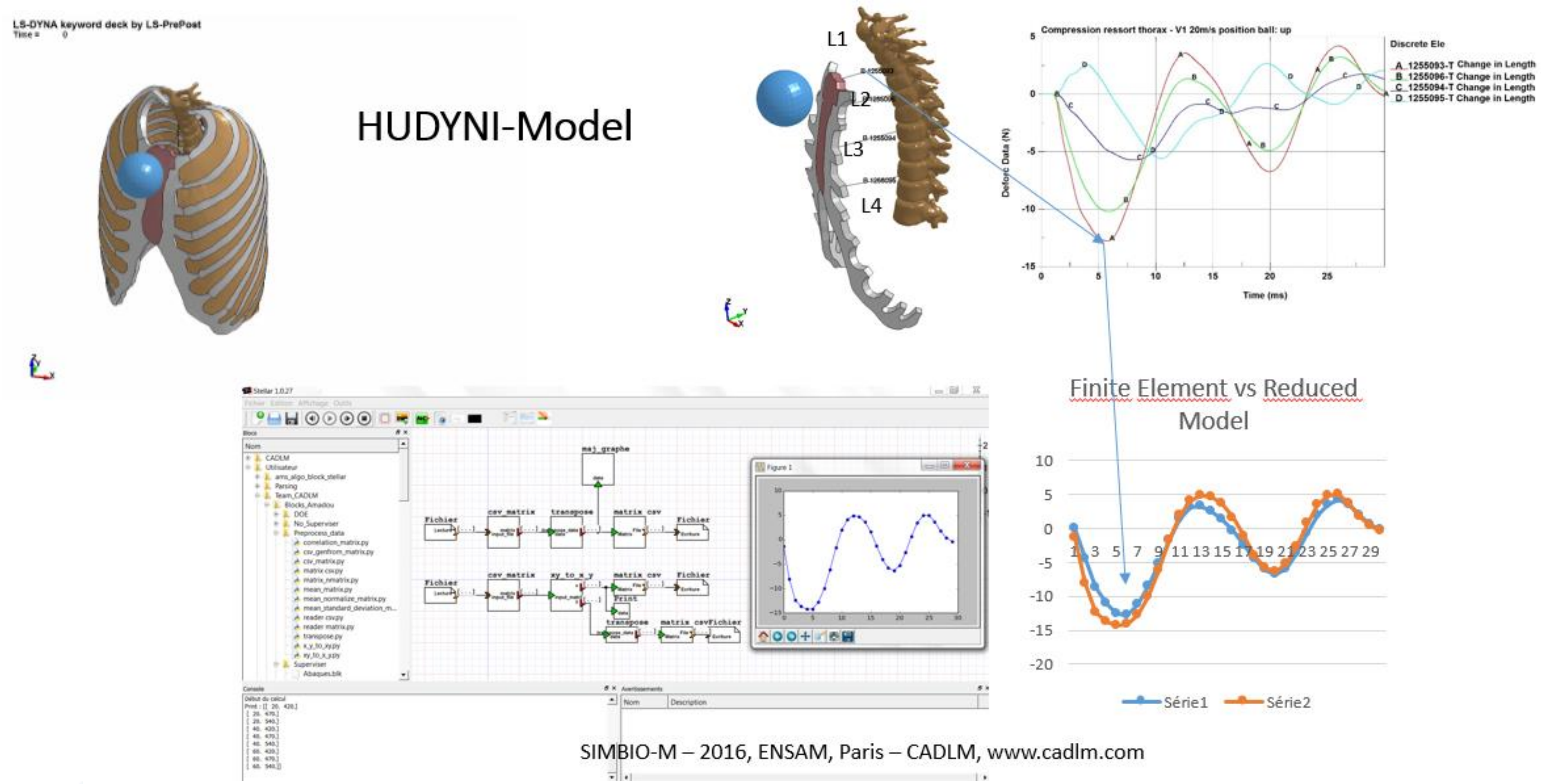
100 FE results



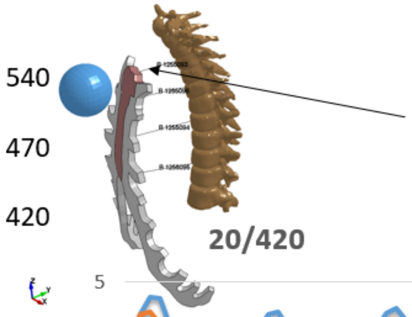
10000 reduced mode results

Courtesy of
Toyota Motor Company,
Tsuyoshi Yasuki,
LS-DYNA conference 2018

Example – Morphing + (personalized impact on thorax)

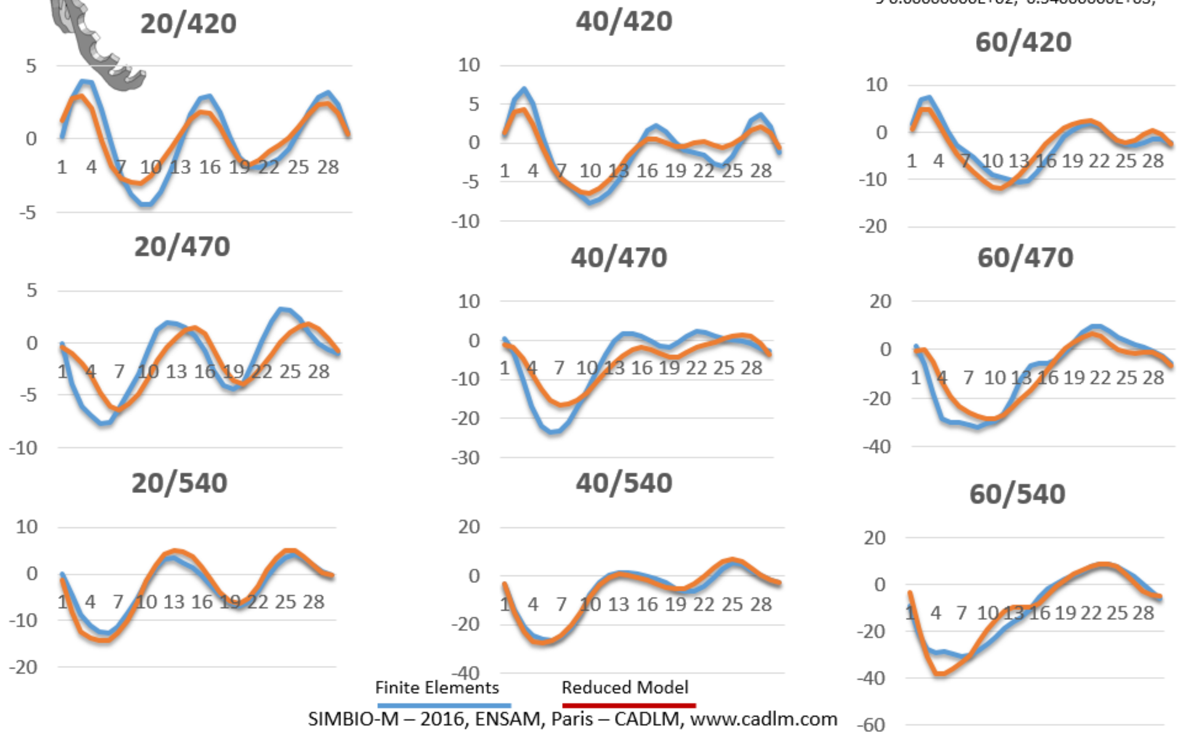


Biomechanics - Predicted results



L1 / Réponse

Impact velocity	Impact position
1 0.20000000E+02;	0.42000000E+03;
2 0.20000000E+02;	0.47000000E+03;
3 0.20000000E+02;	0.54000000E+03;
4 0.40000000E+02;	0.42000000E+03;
5 0.40000000E+02;	0.47000000E+03;
6 0.40000000E+02;	0.54000000E+03;
7 0.60000000E+02;	0.42000000E+03;
8 0.60000000E+02;	0.47000000E+03;
9 0.60000000E+02;	0.54000000E+03;



SIMBIO-M – 2016, ENSAM, Paris – CADLM, www.cadlm.com

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